Int J Thermophys (2008) 29:483–504
DOI 10.1007/s10765-008-0433-x

A New Method for Liquid Viscosity Measurements: Inclined-Tube Viscometry

Y. Zhang · M. G. He · R. Xue · X. F. Wang · Q. Zhong · X. X. Zhang

Received: 11 January 2007 / Accepted: 2 April 2008 / Published online: 1 May 2008

Abstract In this article, a method which can be used to measure the viscosity of liquids with an inclined tube at high pressures and for low-boiling substances is described. The measurement equation was established. The measuring methods for two unknown parameters which are in the measurement equation are presented, and a viscosity measurement system was designed and constructed, which consists of an experimental cell, an inclination-angle control subsystem, a constant temperature subsystem, and a data collection and process subsystem. At atmospheric pressure, the kinematic viscosity of pure water was measured at temperatures from 273.15 K to 333.15 K to demonstrate the performance of the apparatus. The results show that the absolute average relative deviation is 0.84% in comparison with reliable literature values. The kinematic viscosity of saturated liquid R134a and R600a were also measured at temperatures from 273.15 K to 295.15 K and 273.15 K to 300.15 K, respectively, and the corresponding absolute average relative deviations are 1.04% and 1.02% in comparison with reliable literature values. These experimental results demonstrate the performance of the apparatus, while providing estimates of the uncertainty and reliability of the experimental system.

Keywords Inclined tube · Pure water · R134a · R600a · Viscosity

Nomenclature

g Acceleration of gravity
l Length of the liquid column
Q Fluid flow

Y. Zhang · M. G. He (✉) · R. Xue · X. F. Wang · Q. Zhong · X. X. Zhang
State Key Laboratory of Multiphase Flow in Power Engineering, Xi'an Jiaotong University, Xi'an Shaanxi 710049, People's Republic of China
e-mail: mghe@mail.xjtu.edu.cn

R Pipe inside radius
r Integration radius
u_0 Velocity of the liquid column obtained from the practical measurement when the liquid column flows at a uniform speed
u Velocity of the liquid column

Greek letters

α Angle between the direction of the thin tube and horizontal direction
γ Surface tension acting on liquid
η Dynamic viscosity of the experimental liquid
θ Angle
π Circumference ratio
ρ Density of the experimental liquid
τ Shearing stress
ν Kinematic viscosity

Subscripts

s Standard liquid
c,b Contact angle of back meniscus
c,f Contact angle of front meniscus

1 Introduction

Viscosity is a measure of the internal fluid friction that tends to oppose any dynamic change in the fluid motion; the quantitative relationship between viscosity and internal fluid friction is given by Newton's law of viscosity which states that viscosity is the ratio of the shearing stress to the rate of shear in steady-state flow [1,2]. In many fields of the national economy such as the petroleum industry, chemical industry, food industry, building materials, coal industry, metallurgical industry, cosmonautics, and medicine, the viscosity and its measurement are widely used. Viscosity measurements are important for production flow control, guarantee of the production safety, control and evaluation of product quality, medical diagnosis, and scientific research. According to the different measuring principles, traditional viscometry can be grouped as follows: capillary viscometry [3–8], rotating viscometry [9–11], falling ball or needle viscometry [12–15], vibration viscometry [16–19], etc.

Among the multitudinous techniques, capillary viscometry is attractive and widely used in experimental research and engineering practice, and it has been used to establish the viscosity standard because of its relatively high measurement accuracy [7]. However, traditional capillary viscometry has several drawbacks in practical applications. First, the measurement equation requires a kinetic-energy adjustment and an end adjustment [1,2]. Second, there are surface tensions acting on the two liquid levels in the experimental cell; when the conditions of the two meniscuses are different, the surface tensions acting on the two meniscuses will not be equal, and the measurement

accuracy will be impacted [1,2,4,20]. Third, the viscosity can only be measured at one specific shear rate at a time. When the viscosity of a non-Newtonian fluid is measured, since the apparent viscosity of a non-Newtonian fluid can only be obtained by conducting measurements at different shear rates, the level of effort will be greatly increased [20–22]. Fourth, the position of the meniscus is usually read by the naked eye, and although the use of a probe sensor avoids the inaccuracy inherent to the naked eye, the state of the meniscus is altered because of the contact of the probe with the liquid surface, and the measurement results are impacted. Fifth, when the viscosity of a macromolecular solution is measured, because of the effect of the polar force, the macromolecule which is in the macromolecular solution will not only adsorb on the capillary wall to reduce the capillary's effective diameter, but also lead to significant variations of liquid–solid boundary properties and the measurement results will be impacted [23,24]. If the capillary diameter is too small, in the process of measuring the viscosity of blood and suspending liquid, the diameter effect which can also be called the $\sum$ effect will occur [20]. For other viscosity measurement techniques, there are similar deficiencies which are not given detailed descriptions here in consideration of the length of this article.

Due to the reasons given above, the development of new methods for liquid viscosity measurements is desired. In recent years, with the development of the electronic industry and computer technology, a large number of electronic devices and computer control systems, both of which provide the necessary motivation for the development of viscometry with high accuracy, high efficiency, and high automation, have gradually been applied to viscosity measurements. The newly developed methods for liquid viscosity measurements include electromagnetic induction viscometry and supersonic wave Doppler viscometry, both of which are examples of the application of electronic sensors to liquid viscosity measurements.

In this article, a new method to measure the viscosity of liquids, namely, "the inclined-tube viscometer," has been proposed and the measurement principle of the method has been established. Then a new experimental cell has been designed and developed, which can be used to measure the viscosity of liquids at atmospheric pressure, that of liquids at high pressures, and for low-boiling substances in the liquid phase. In order to test and verify the measuring principle, the kinematic viscosity of pure water was measured at 293.15 K at atmospheric pressure. In order to verify the accuracy and reliability of the experimental system at other temperatures, the kinematic viscosities of refrigerants R134a and R600a were also measured in the saturated liquid phase. On the basis of the experimental results, the experimental principle and apparatus were analyzed, and the potential improvement of this viscometer for future work is presented.

2 Theory

As shown in Fig. 1, a length of liquid column flows at a uniform speed along the axial direction in a thin tube which is oblique to a horizontal surface. The flow of the liquid column's inner fluid is a fully developed isothermal laminar flow. The pressures of both meniscuses are equal. The component of the liquid column is an incompressible

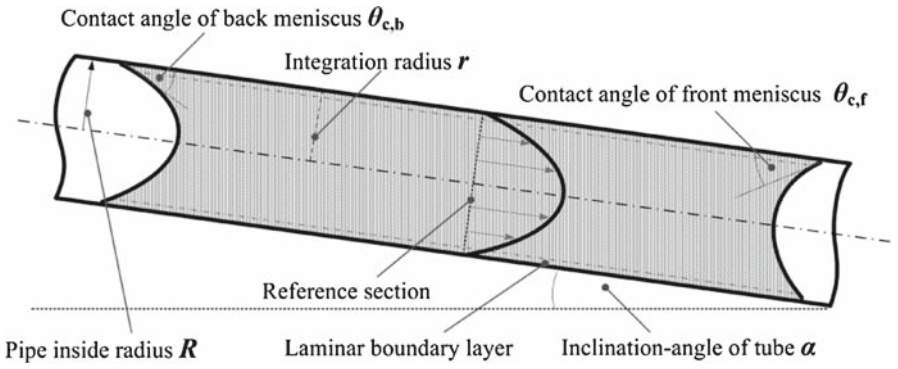

Fig. 1 Flow model of the liquid column in the tube

fluid. The flow of the liquid column that contacts the tube inner wall is non-slip flow. The thin tube is long and straight enough, and its inner diameter is uniform.

Based on the above assumptions, there are four forces acting on the liquid column in the direction of its flow. The first force is the component force of the liquid column's gravity along the axial direction of the tube. The second force is the surface tension acting on the front meniscus. The third force is the surface tension acting on the back meniscus. The fourth force is the viscous resistance of the fluid. These four forces are in dynamic equilibrium when the liquid column flows at a uniform speed in the tube. (Here, it should be reported that the effect of the buoyancy force has also been considered when establishing the measurement principle. Via experimental research, it can be seen that the effect of the buoyancy force on the liquid column can be neglected. In future studies, this effect may be added to the measurement equation as an adjustment coefficient.)

Considering the above forces, the equilibrium equation describing the uniform flow of the liquid column can be expressed as

$$\pi r^2 l\rho g \sin\alpha + \frac{2\pi R\gamma(\cos\theta_{c,b} - \cos\theta_{c,f})}{\pi R^2} \cdot \pi r^2 = 2\pi r l\tau \tag{1}$$

In this equation, ρ is the density of the liquid sample; α is the angle between the direction of the thin tube and the horizontal direction, which can be called the thin tube inclination angle; R is the inside radius of the thin tube; γ is the surface tension acting on the liquid; $\theta_{c,b}$ is the contact angle of the back meniscus; $\theta_{c,f}$ is the contact angle of the front meniscus; l is the length of the liquid column; τ is the shearing stress; and r is the integration radius.

The term $2\pi r l\tau$ represents the viscous resistance of the liquid column when it flows; the term $\pi r^2 l\rho g \sin\alpha$ represents the component force of the liquid column's gravity along the axial direction of the tube; and the term $2\pi R\gamma(\cos\theta_{c,b} - \cos\theta_{c,f})\pi r^2/\pi R^2$ represents the resultant force formed by the action of surface tensions of both menisCuses, and it acts as a pressure and this term can be obtained from the surface energy equation [25,26]. Equation 1 can be applied not only to a Newtonian fluid but also to a non-Newtonian fluid.

Considering a Newtonian fluid, according to Newton's law of viscosity, an alternative form of Eq. 1 is

$$\pi r^2 l\rho g \sin\alpha + \frac{2\pi R\gamma(\cos\theta_{c,b} - \cos\theta_{c,f})}{\pi R^2} \cdot \pi r^2 = -2\pi r l\eta \cdot \frac{du}{dr} \tag{2}$$

Assume:

$$A = \left[\rho g l \sin\alpha + \frac{2\gamma\left(\cos\theta_{c,b} - \cos\theta_{c,f}\right)}{R}\right]/l$$

Equation 2 can be written as

$$A \cdot r = -2\eta \cdot \frac{du}{dr} \tag{3}$$

Separating the variables and considering the boundary condition ($u = 0$, when $r = R$), we find

$$u = \frac{A}{4\eta} \cdot \left(R^2 - r^2\right) \tag{4}$$

The fluid flow on the reference section of the thin tube is

$$Q = \int_0^R 2\pi r u \cdot dr \tag{5}$$

Using Eqs. 4, 5, can be written as

$$Q = \frac{A\pi}{8\eta} \cdot R^4 \tag{6}$$

From a practical measurement viewpoint, the fluid flow in the reference section of the thin tube can be expressed as

$$Q = \pi R^2 u_0 \tag{7}$$

In Eq. 7, u_0 is the velocity of the liquid column when the liquid column flows at a uniform speed. Thus, we obtain the following relation,

$$\frac{A\pi}{8\eta} \cdot R^4 = \pi R^2 u_0 \tag{8}$$

Equation 8 can also be written as follows:

$$\left[\rho g l \sin\alpha + \frac{2\gamma\left(\cos\theta_{c,b} - \cos\theta_{c,f}\right)}{R}\right] \cdot R^2/8\eta l = u_0 \tag{9}$$

The equation describing the dynamic viscosity of the liquid which is the component of the liquid column can be deduced from Eq. 9;

$$\eta = \frac{\rho g R^2 \sin\alpha}{8u_0} + \frac{2\gamma R\left(\cos\theta_{c,b} - \cos\theta_{c,f}\right)}{8lu_0} \tag{10}$$

In this equation, there are three unknown parameters which are γ, $\cos\theta_{c,b}$, and $\cos\theta_{c,f}$ in the term $2\,\gamma(\cos\theta_{c,b} - \cos\theta_{c,f})$ which represents the resultant force of the surface tensions. There are difficulties in determining these three parameters, so some methods are adopted to simplify their determination. Through observations of the images of both the front and back meniscus using a high-accuracy CCD camera when the liquid column is in a steady-state-flow condition, it can be concluded that when the liquid column flows at a low speed, the shapes of the front and back meniscuses are approximately the same. The shapes of the two meniscuses vary little as the velocity changes and so it can be considered that the surface tensions acting on the two meniscuses of the liquid column are approximately equal. Considering the numerical values of terms which are on the right side of the equation, the numerical value of the first term is much higher than that of the second term. So the numerical value of the second term which represents the resultant force of the surface tensions can be regarded as a constant.

From what has been discussed above, we assume

$$k_1 = \frac{\rho g R^2}{8\eta}, \quad k_2 = \frac{2\gamma R\left(\cos\theta_{c,b} - \cos\theta_{c,f}\right)}{8l\eta}$$

and according to Eq. 10, the following relation can be obtained:

$$u_0 = k_1 \cdot \sin\alpha + k_2 \tag{11}$$

From Eq. 11, a conclusion that can be reached is that when the tube has different inclination angles and the liquid column flows at a uniform speed, as long as the velocity of the liquid column and the value of the inclination angle are known, a linear curve can be fitted by least squares and the rate of the slope of this curve is k_1. Ultimately, the kinematic viscosity ν of the experimental liquid can be determined from the equation describing k_1, if the inner radius R and local acceleration of gravity g are known in advance. The dynamic viscosity η of the experimental liquid can also be determined if the density ρ is known in advance.

Table 1 lists the equations for the viscosity measurements of a Newtonian fluid using the absolute measuring method and the relative measuring method, and subscript s represents the standard liquid. When absolute measurement equations are adopted, if the inner radius R of the thin tube is known, and a series of inclination angles of the tube α and the corresponding values of the uniform speed of the liquid column u are measured, the viscosity of the experimental liquid can be determined. When relative measurement equations are adopted, the value of k_{1s} of the standard liquid whose viscosity is known should be first measured, and the value of k_1 of the test liquid should be measured using the same thin tube that is used in measuring the value

Table 1 Equations for viscosity measurement of Newtonian fluid using Inclined-tube viscometry

Absolute measurement equation		Relative measurement equation	
Dynamic viscosity	Kinematic viscosity	Dynamic viscosity	Kinematic viscosity
$\eta = \rho g R^2/8k_1$	$\nu = gR^2/8k_1$	$\eta = \rho k_{1s}\eta_s/\rho_s k_1$	$\nu = k_{1s}\nu_s/k_1$

Note: In Xi'an, the acceleration of gravity is $g_{XA} = 9.795\,\mathrm{m\cdot s^{-2}}$

of k_{1s}. When measuring the dynamic viscosity, the density of the experimental liquid and that of the standard liquid should be known.

3 Development of Experimental Apparatus and System

According to the measuring principle introduced above, a new experimental cell was designed and developed (shown in Fig. 2). The experimental cell can be used to measure the viscosity of liquids at atmospheric pressure, that of liquids under high pressures, and that for low-boiling substances in the liquid phase. The experimental cell primarily consists of a measuring unit for the velocity of the liquid column, an experimental glass tube, a holder, an injection device, and a constant pressure cylinder. The measuring unit for the velocity consists of four groups of optical fiber sensors. Using a digital vernier caliper, the four groups of optical fiber sensors were accurately located on the holder along the axial direction of the experimental glass tube. For the optical fiber sensor groups, their operating principle for measuring the velocity is described as follows. When the front meniscus of the liquid column passes through the optical fiber sensor, the sensor will give out a step signal. When the distance between two adjacent sensors are determined in advance, by recording the time difference of the step signal given by the two adjacent sensors, the average velocity of the liquid column passing through the two adjacent sensors can be determined. Three average velocities of the liquid column can be obtained by setting four groups of sensors. Accordingly, whether the liquid column flows at a uniform speed can be estimated and this uniform speed can be recorded. From experimental observation, it can be seen that from the beginning of flow to the state of uniform speed, the accelerated distance of the liquid column is very short. So by setting the sensor group properly, the three velocities, or at least the last two velocities, can be equal. This equal velocity is the velocity of the liquid column flowing at a uniform speed.

Considering the optical measurements, the experimental glass tube is chosen to be a thin tube. Considering convenient monitoring of the flow pattern of the liquid column in the experimental glass tube, the constant pressure cylinder is made of plexiglass. The injection device is composed of a liquid storage reservoir, and the valves are shown on the left side of Fig. 2. By controlling the valves, a length of liquid column of the experimental liquid can be obtained in the experimental glass tube. After the injection, the holder is pulled through the right-hand handle and the connection between the left side of the experimental glass tube and injection device is broken. This operation can avoid discontinuance of the liquid column in the glass tube, and the pressure on both meniscuses of the liquid column will be equal. The temperature in the constant

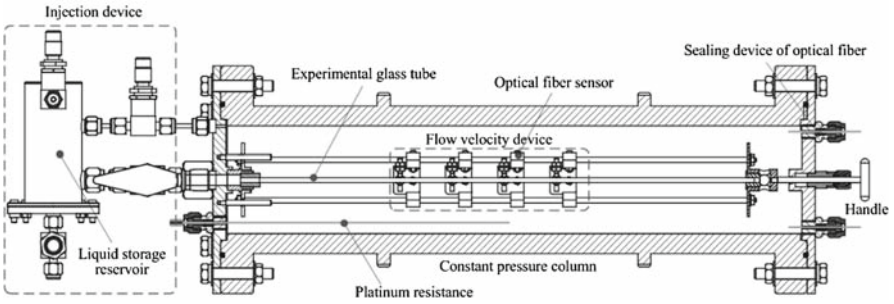

Fig. 2 Experimental cell of inclined-tube viscometer

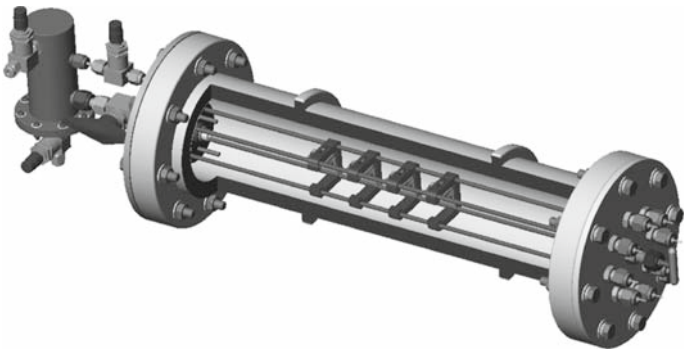

Fig. 3 Three-dimensional model of experimental cell

pressure cylinder is measured with a platinum resistance thermometer, Pt100. The real-time detection of the pressure in the cylinder is achieved with a pressure sensor connected to the opening on the right-hand flange of the cylinder as shown in Fig. 2. Figure 3 depicts a three-dimensional model of the experimental cell.

Figure 4 shows the rotation stage used in this work. By the corresponding connection unit, the experimental cell was fixed on the rotation stage whose rotational angle and rotational speed can be set by the corresponding control unit.

In order to carry out the experiment at a stable ambient temperature, a constant temperature cabinet using air as the working fluid has been designed (shown as Fig. 5). A circulating air duct where a heating device and an evaporator are fixed is located in this constant temperature cabinet. The temperature in the working body ($900 \times 700 \times 600\,\mathrm{mm}^3$) can be adjusted from 265.15 K to 573.15 K. The experimental results show that the uncertainty of the temperature control in the working body is 0.3 K for temperatures from 273.15 K to 423.15 K. When the experimental cell and rotation stage were placed in the working body to test the temperature control at temperatures from 273.15 K to 423.15 K, it can be found that the temperature fluctuations in the constant-pressure cylinder of experimental cell is within ± 0.2 K for periods longer than 30 min. (A 30 min period is much longer than the time needed for a single experiment)

Figure 6 shows the data collection and processing system. The system includes a chassis (SCXI-1000), a 32-channel analog input module (SCXI-1102), a current

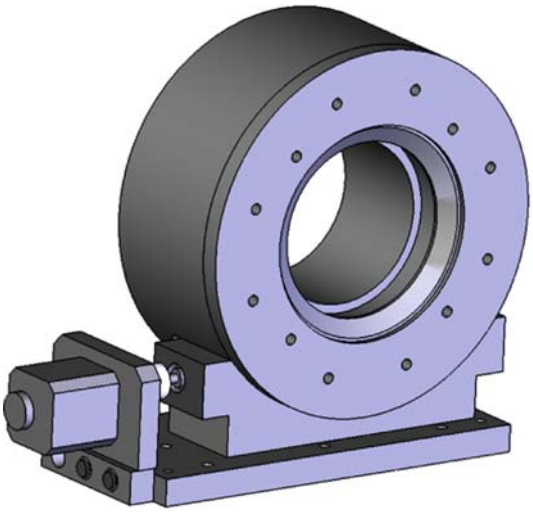

Fig. 4 Electrical rotation stage control

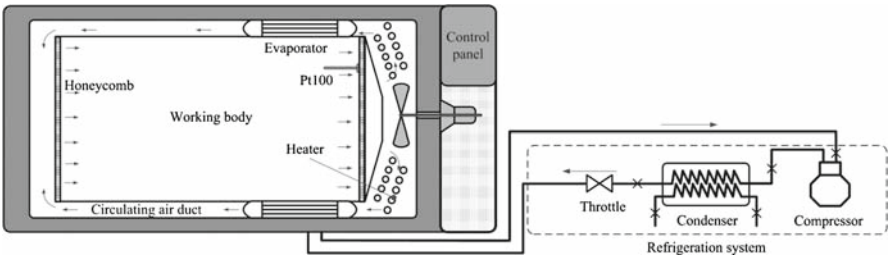

Fig. 5 Constant temperature cabinet with air as working fluid

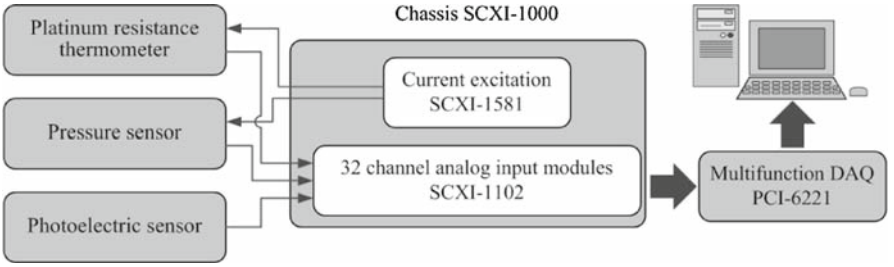

Fig. 6 Data collection and processing system

generator (SCXI-1581), a multifunction DAQ (PCI-6221), and a PC. Connected to the platinum resistance thermometer, pressure sensor, and the group of optical fiber sensors, the system, controlled by a corresponding measurement program written in LABVIEW, can be used to measure the temperature, pressure, and the velocity of the liquid column.

4 Measurement Process

Figure 7 shows a flow process diagram of the measurement. Before initiation of experiment, the following should be considered. First, the experimental glass tubes with different inside radius should be chosen according to the viscosity of the experimental liquid and the quantity of liquid sample needed in the experiment should be estimated. Second, when the viscosity of macromolecular solution is measured, in order to avoid adsorption, the experimental tubes that are made of different materials should be chosen according to the nature of the solution. Last, when the viscosity of a transparent liquid is measured, in order to make the observation more convenient, the liquid may be colored in advance as long as the addition of color does not change the properties of the sample.

The measurement process primarily includes the following five steps: First, the experimental glass tube should be chosen. The experimental glass tube should be sufficiently long and straight, and its inner diameter should be uniform. When the liquid flows in the glass tube, the shapes of both meniscuses of the experimental liquid are related to the inside diameter of the glass tube, and accordingly the applicability of the measurement equation will be affected. As a result, glass tubes with different inside diameters are chosen according to the viscosity of the experimental liquid. The

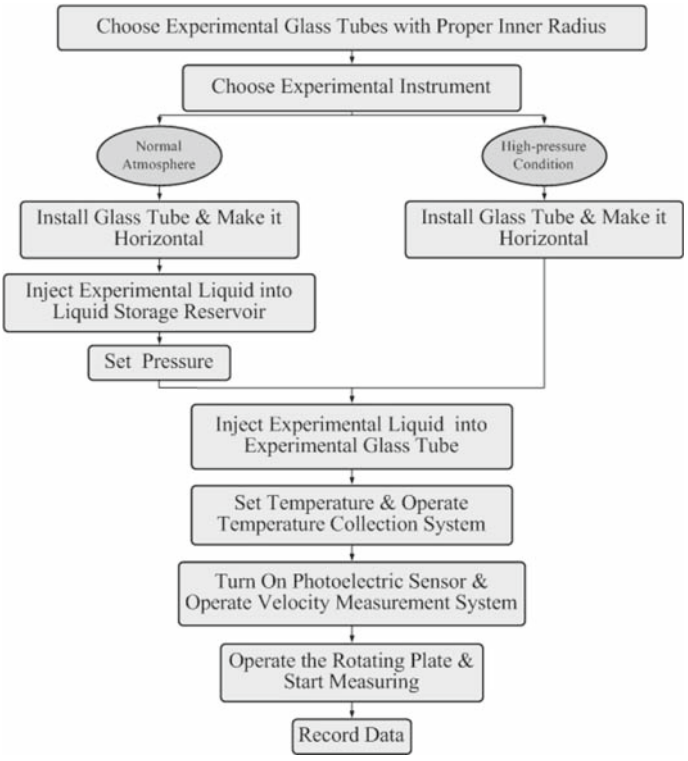

Fig. 7 Measurement process

inside diameter of the selected glass tube should vary directly with the viscosity of the liquid sample.

Second, the liquid storage reservoir of the experimental cell should be evacuated. Then the experimental liquid is injected into the liquid storage reservoir. Next, the liquid storage reservoir gas valve that connects with the sleeve body of the experimental cell is opened. As there is a pressure difference between the liquid storage reservoir and the sleeve, some experimental liquid will evaporate and the pressure in the working cavity will increase. Then the liquid storage reservoir liquid valve with which the liquid storage reservoir and glass tube are connected is opened. After that, the residual experimental liquid is injected into the glass tube and a length of liquid column is formed. After equilibrium conditions are attained, the viscosity of the liquid at saturation is measured. The system needs an external pressurization device if the viscosity at pressures above saturation is measured. When the length of the liquid column satisfies the measurement requirements, the connection between the glass tube and liquid reservoir is opened, and the two ends of the liquid column will be connected in the working cavity by adjusting the location of the holder for the glass tube along the axial direction of the glass tube, and equilibrium of pressures at both ends of the liquid column is attained. The whole experimental cell that is operated at high pressure should be fixed on the inclination-angle control subsystem and be set level before the injection of the liquid.

When the measurements are carried out at atmospheric pressure, the apparatus shown in Fig. 8 can be used. The experimental liquid can be brought in directly through the experimental glass tube.

Third, the whole viscosity measurement system is placed inside the constant temperature cabinet. The test temperature is adjusted with the temperature controller. The temperatures around the glass tube are measured with a platinum resistance thermometer that is fixed on the holder for the glass tube. Since there is some liquid in the glass tube and the wall of the glass tube is thin, the temperature measured with the platinum resistance thermometer is assumed to be the temperature of the experimental liquid. After the temperature reaches equilibrium, the glass tube is slanted clockwise and anti-clockwise several times using the rotating plate controller, and the experimental liquid flows repeatedly in the glass tube. Accordingly, the inner wall of the

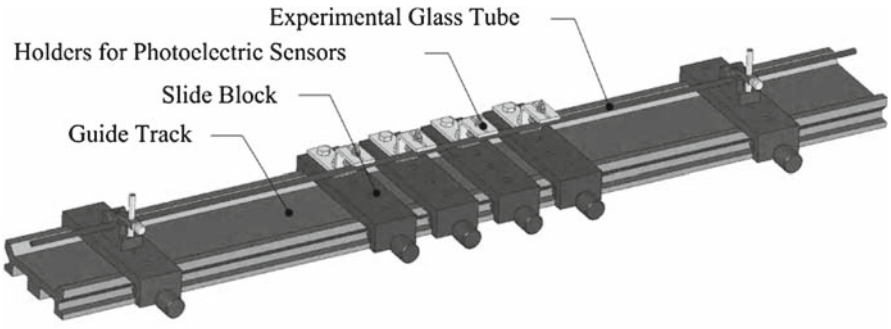

Fig. 8 Structural pattern of viscosity measurement device operated at atmospheric pressure

glass tube will be fully wetted by the experimental liquid and instability of the liquid column flow pattern will be avoided.

Fourth, after a steady flow pattern of the liquid column is established, the liquid column should be placed to one side of the glass tube and the photoelectric sensors and velocity collection system are turned on. Then the glass tube is slanted to a selected inclination angle, and the liquid column will flow in the glass tube and reach a uniform speed after acceleration in a short length. Then, using the photoelectric sensors, the uniform speed of the liquid column is measured as a function of the inclination angle of the glass tube. The viscosity is then determined using the slope of the linear curve between the velocity of the liquid column and the inclination angle.

The drawbacks of a traditional capillary viscometer, including the simplification of the calculation formula and the low measurement accuracy caused by the deficiency in designing such a device, are avoided effectively by the method for liquid viscosity measurements presented in this article. This method has advantages such as a short measurement period, high measurement accuracy, and simple operation. It needs only a small amount of liquid sample, and it can be easily automated.

5 Analysis of Experimental Uncertainty

According to the guidelines for expressing the experimental uncertainty given by the International Organization for Standardization, the total or expanded uncertainty, U, describing the temperature, the pressure, and the viscosity, can be calculated from the following equation.

$$U = ku_{\mathrm{c}} = k\sqrt{\sum (u_{\mathrm{i}})^2} \tag{12}$$

where u_i is the standard deviation of each error source; u_c is the combined standard deviation including all sources of error; and k is the coverage factor, which is usually taken to be two or three. When $k = 2$, the level of confidence is 95%; when $k = 3$, the level of confidence is 99%. In this study, the coverage factor is taken to be two.

The total or expanded experimental uncertainties in temperature, pressure, and viscosity are estimated to be not greater than ± 0.24 K, ± 3.72 kPa, and ± 0.64%, respectively (See Table 2).

6 Performance Tests of the Experimental Principle and Apparatus

6.1 Measurements of the Kinematic Viscosity of Pure Water at 293.15 K

At atmospheric pressure, the kinematic viscosity of pure water was independently measured eight times at 293.15 K. Table 3 lists the experimental results and their absolute and relative deviations compared with values calculated from NIST-REFPROP 7.0. From Table 3, it can be seen that the maximum relative deviation is -1.09%, the absolute average of the relative deviations is 0.85%, the average of the eight experimental results is 1.0027 $\mathrm{mm^2 \cdot s^{-1}}$, and the relative deviation is only 0.01% compared with

Table 2 Experimental uncertainties for temperature, pressure, and viscosity

Temperature	
Platinum resistance thermometer, u_1	0.01 K
Data collection and process detector equipment, u_2	0.025 K
Temperature control system, u_3	0.05 K
Temperature stability of constant temperature cabinet, u_4	0.10 K
Combined standard uncertainty, u_c	0.12 K
Pressure	
Pressure sensor, u_1	1.86 kPa
Data collection and process detector equipment, u_2	0.05 kPa
Combined standard uncertainty, u_c	1.86 kPa
Viscosity	
Speed of the liquid column, u_1	0.27%
Inclination-angles of the experimental glass pipe, u_2	0.11%
Inside radius of the experimental glass pipe, u_3	0.13%
Combined standard uncertainty, u_c	0.32%

Table 3 Kinematic viscosity of pure water at 293.15 K and at normal atmospheric pressure

	ν ($mm^2 \cdot s^{-1}$)[a]	ν' ($mm^2 \cdot s^{-1}$)[b]	$(\nu - \nu')$ ($mm^2 \cdot s^{-1}$)	$(\nu - \nu')/\nu'$(%)
	1.0113		0.0087	0.87
	0.9996		−0.0030	−0.30
	1.0124		0.0098	0.98
	0.9917		−0.0109	−1.09
	0.9932	1.0026	−0.0094	−0.94
	1.0103		0.0077	0.77
	1.0107		0.0081	0.81
	0.9926		−0.0100	−1.00
Average	1.0027		0.0001	0.01

Absolute average of relative deviations: 0.85%
[a] ν is the experimental result
[b] ν' is the literature value [27]

the calculated values from REFPROP. From Fig. 9, it can be seen that the average deviations between experimental results and reference data are basically concentrated in an area of $\pm 0.015\,mm^2 \cdot s^{-1}$. Thus, the experimental system has a relatively good reproducibility.

Table 4 lists the values of the inclination angle and the corresponding uniform speeds of the liquid column, where the experimental result for the kinematic viscosity of pure water is $0.9996\,mm^2 \cdot s^{-1}$ at 293.15 K and atmospheric pressure. In this experiment, the range of the inclination angle is (0.70–1.20)°, and the gap is 0.05°. In order to test the reproducibility of the corresponding relation between the liquid column's uniform speed and the inclination angle, the inclination angle adopted in this measurement was first increased with a constant gap, and then decreased. From Table 4, it can be seen that there is good reproducibility of the uniform speed. The value of k_1, derived from the values of this group of speeds and their corresponding inclination angles, is 491.72. The value of k_2 is −0.13. The value of the term $k_1 \cdot \sin\alpha$ is much larger than that of k_2, and this result is in agreement with the analysis in Sect. 2. Figure 10

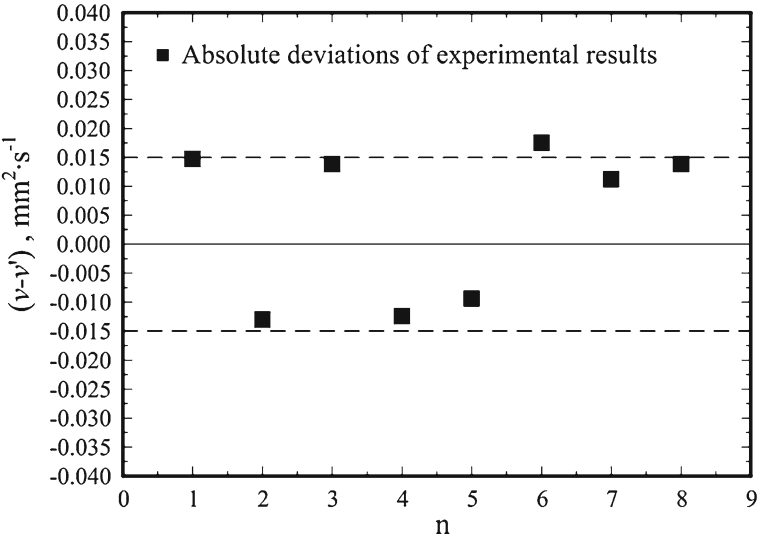

Fig. 9 Absolute deviations of experimental results of the kinematic viscosity of pure water at normal atmospheric pressure and 293.15 K

Table 4 Inclination angles and their corresponding velocities

Inclination angle increases		Inclination angle decreases		$(u_0 - u_0')$	k_1	k_2
α (°)	u_0 (cm·s^{-1})	α (°)	u_0' (cm·s^{-1})	(cm·s^{-1})		
0.70	5.8612	0.70	5.9101	−0.0489		
0.75	6.3527	0.75	6.3124	0.0403		
0.80	6.7342	0.80	6.7452	−0.0110		
0.85	7.1458	0.85	7.1634	−0.0176		
0.90	7.6073	0.90	7.5987	0.0086		
0.95	8.0451	0.95	8.0099	0.0352	491.72	−0.13
1.00	8.4320	1.00	8.4445	−0.0125		
1.05	8.8518	1.05	8.8537	−0.0019		
1.10	9.2669	1.10	9.2749	−0.0080		
1.15	9.8074	1.15	9.7614	0.0460		
1.20	10.216	1.20	10.1786	0.0374		

depicts the curve fitted by the velocity of liquid column and the sinusoidal value of the inclination angle.

6.2 Measurements of the Kinematic Viscosity of Pure Water, Refrigerant R134a, and Refrigerant R600a

In order to further test the reliability of the measuring principle and the experimental system at normal atmospheric pressure, the kinematic viscosity of pure water was measured at temperatures from 273.15 K to 333.15 K. Then the kinematic viscosities of refrigerant R134a (at temperatures from 273.15 K to 295.15 K) and refrigerant R600a

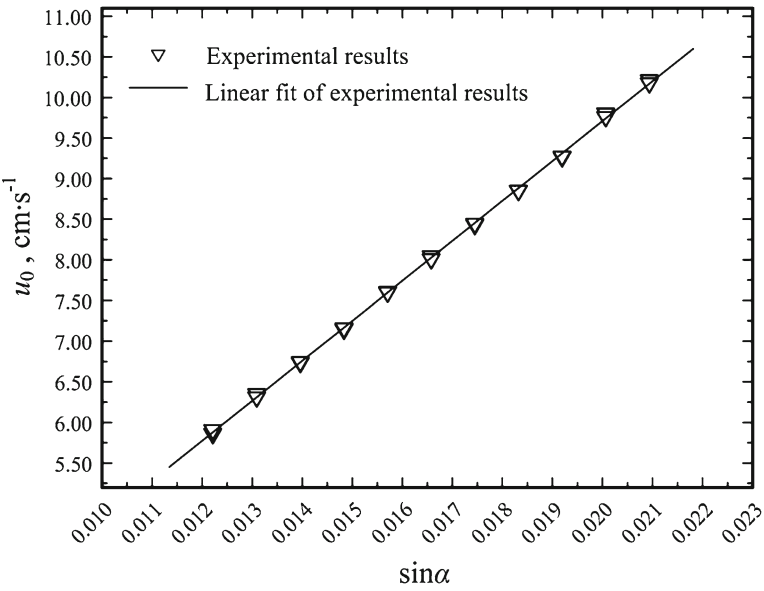

Fig. 10 Curve fitted by the velocity of liquid column and the sinusoidal value of the inclination angle when the experimental result is 0.9996 $mm^2 \cdot s^{-1}$

(at temperatures from 273.15 K to 300.15 K) for the saturated-liquid phase were also measured.

Table 5 and Figs. 11 and 12 show the experimental results and literature values for the viscosity of pure water and the deviations between them. It can be seen that the maximum relative deviation is 1.25% compared with literature values, the absolute average of the relative deviations is 0.84%, and the absolute deviations of all the data points cover a range from $-0.006\,mm^2 \cdot s^{-1}$ to $0.014\,mm^2 \cdot s^{-1}$.

Table 6 and Figs. 13 and 14 show the experimental results and literature values for the viscosity of refrigerant R134a and the deviations between them. It can be seen that the maximum relative deviation is 1.14% compared with literature values, the absolute average of relative deviations is 1.04%, and the absolute deviations of all the data points cover a range from $-0.003\,mm^2 \cdot s^{-1}$ to $0.003\,mm^2 \cdot s^{-1}$.

Table 7 and Figs. 15 and 16 show the experimental results and literature values for the viscosity of refrigerant R600a and the deviations between them. It can be seen that the maximum relative deviation is 1.46% compared with literature values, the absolute average of relative deviations is 1.02%, and the absolute deviations of all data points cover a range from $-0.004\,mm^2 \cdot s^{-1}$ to $0.005\,mm^2 \cdot s^{-1}$.

6.3 Discussion

In the process of deducing the final measuring equation, some simplifications were applied to the equation describing the flow of the liquid column that has a uniform speed. The main conditions of the simplifications are that the surface tensions acting

Table 5 Kinematic viscosity of pure water from $T = 274.95\,\mathrm{K}$ to $333.12\,\mathrm{K}$

T (K)	ν ($\mathrm{mm^2 \cdot s^{-1}}$)[a]	ν' ($\mathrm{mm^2 \cdot s^{-1}}$)[b]	$(\nu - \nu')$ ($\mathrm{mm^2 \cdot s^{-1}}$)	$(\nu - \nu')/\nu'$ (%)
274.95	1.6975	1.6848	0.0127	0.75
277.91	1.5429	1.5297	0.0132	0.86
282.73	1.3317	1.3222	0.0095	0.72
284.52	1.2511	1.2565	−0.0054	−0.43
289.23	1.1125	1.1069	0.0056	0.51
293.10	1.0117	1.0046	0.0071	0.71
297.56	0.9141	0.9047	0.0094	1.04
300.23	0.8601	0.8525	0.0076	0.89
303.82	0.7951	0.7897	0.0054	0.68
306.28	0.7451	0.7510	−0.0059	−0.79
310.54	0.6875	0.6910	−0.0035	−0.51
314.21	0.6513	0.6455	0.0058	0.90
317.95	0.6105	0.6041	0.0064	1.06
321.53	0.5739	0.5684	0.0055	0.97
323.11	0.5599	0.5538	0.0061	1.10
324.89	0.5321	0.5381	−0.0060	−1.12
327.01	0.5232	0.5204	0.0028	0.54
329.09	0.5103	0.5040	0.0063	1.25
330.15	0.5010	0.4960	0.0050	1.01
331.97	0.4881	0.4827	0.0054	1.12
333.12	0.4712	0.4746	−0.0034	−0.72

Absolute average of relative deviations: 0.84%
[a] ν is the experimental result
[b] ν' is the literature value [27]

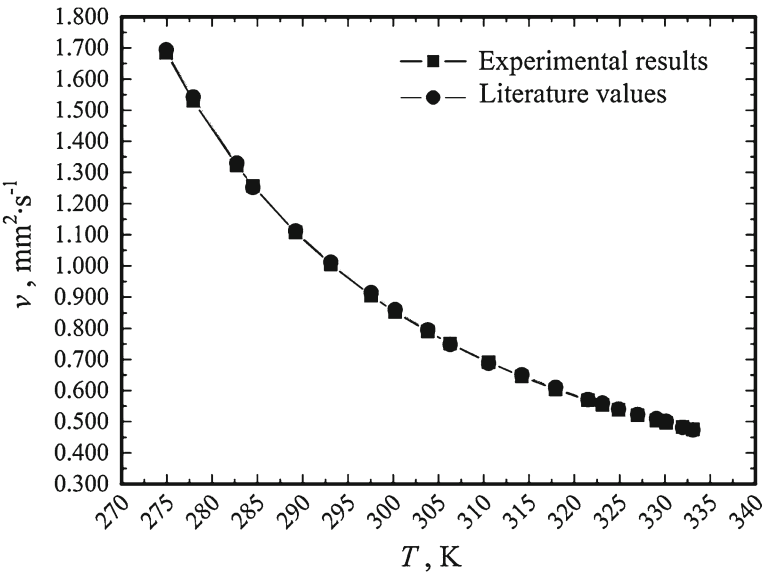

Fig. 11 Comparisons of the experimental results and literature values for pure water

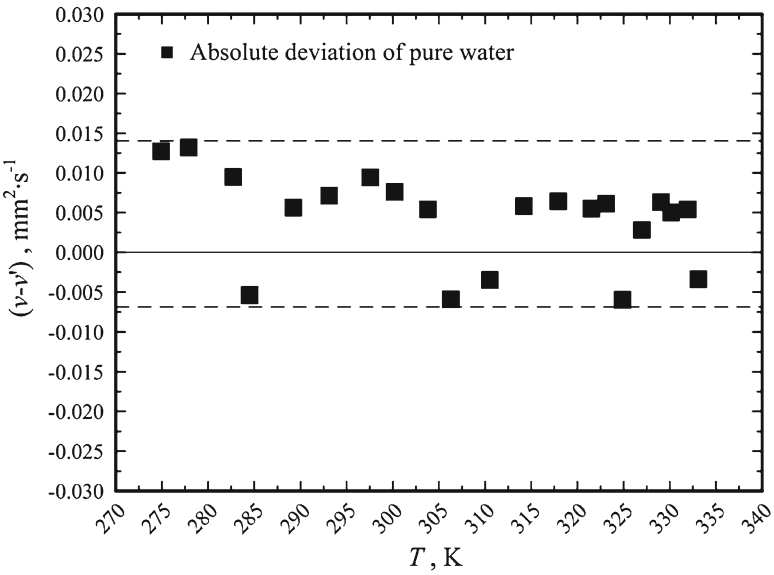

Fig. 12 Distribution of the absolute deviations for measurements of pure water

Table 6 Kinematic viscosity of refrigerant R134a in the saturated liquid phase from $T = 274.73$ K to 294.76 K

T (K)	p_s (MPa)	ν (mm$^2\cdot$s^{-1})[a]	ν' (mm$^2\cdot$s^{-1})[b]	$(\nu - \nu')$ (mm$^2\cdot$s^{-1})	$(\nu\text{-}\nu')/\nu'$ (%)
274.73	0.3104	0.2049	0.2026	0.0023	1.14
275.85	0.3251	0.1981	0.2003	−0.0022	−1.10
278.17	0.3510	0.1976	0.1957	0.0019	0.97
280.43	0.3781	0.1934	0.1913	0.0021	1.10
282.60	0.4072	0.1856	0.1873	−0.0017	−0.91
283.84	0.4244	0.1829	0.1850	−0.0021	−1.14
286.39	0.4609	0.1789	0.1805	−0.0016	−0.89
288.57	0.4948	0.1752	0.1768	−0.0016	−0.90
290.25	0.5220	0.1720	0.1739	−0.0019	−1.09
292.92	0.5651	0.1715	0.1696	0.0019	1.12
294.76	0.6011	0.1685	0.1667	0.0018	1.08

Absolute average of relative deviations: 1.04%

[a] ν is the experimental result

[b] ν' is the literature value [27]

on the two meniscuses of the liquid column are approximately equivalent and the liquid column has an appropriate length. These two conditions can guarantee that, for the numerical values of terms on the left side of Eq. 9, the numerical value of the first term is much larger than that of the second term. The appropriate length of the liquid column can be determined by repeated experiments. As for the three substances used in this study, the lengths of the liquid column are not shorter than 12, 8, and 8 mm, respectively. The intuitive way to guarantee the surface tensions acting on the two meniscuses of the liquid column that are approximately equivalent is by maintaining equivalent shapes of these two meniscuses. Through experimental observations, it can

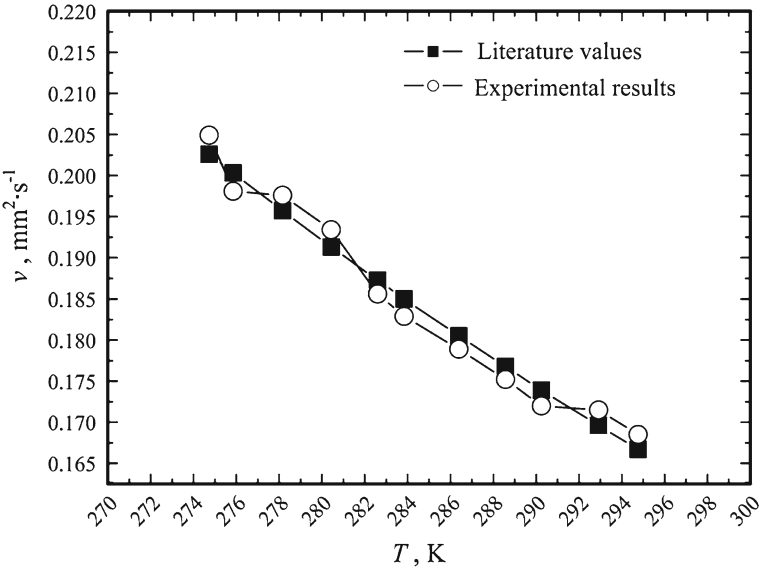

Fig. 13 Comparisons of the experimental results and literature values for R134a

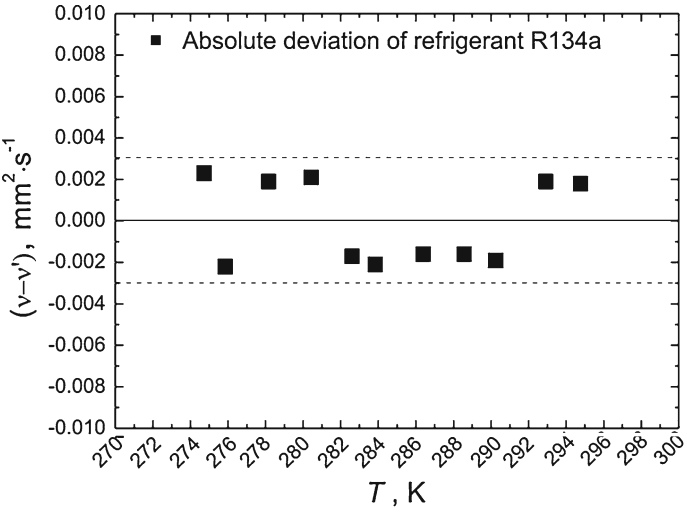

Fig. 14 Distribution of the absolute deviations for measurements of R134a

be found that, of the two meniscuses of the liquid column, the front one is more easily affected.

The main factors to affect the shape of these two meniscuses are the inner radius of the glass tube and the velocity of the liquid column in the measuring process. The liquid will escape from the front meniscus if the inner radius of the glass tube is too large and if the liquid column flows too fast just as Fig. 17 shows. The selection of the inner radius depends on the viscosity of the experimental liquid. As for ordinary

Table 7 Kinematic viscosity of refrigerant R600a in the saturated liquid phase from $T = 273.75$ K to 299.65 K

T (K)	p_s (MPa)	ν (mm$^2\cdot$s^{-1})^a	ν' (mm$^2\cdot$s^{-1})^b	$(\nu-\nu')$ (mm$^2\cdot$s^{-1})	$(\nu-\nu')/\nu'$ (%)
273.75	0.1607	0.3358	0.3398	−0.004	−1.18
275.21	0.1671	0.34	0.3351	0.0049	1.46
277.41	0.1821	0.3317	0.3282	0.0035	1.07
279.12	0.1929	0.3202	0.323	−0.0028	−0.87
281.36	0.2059	0.3134	0.3164	−0.003	−0.95
283.23	0.2212	0.3145	0.311	0.0035	1.13
285.34	0.2362	0.3032	0.3052	−0.002	−0.66
286.96	0.2490	0.3039	0.3008	0.0031	1.03
289.27	0.2681	0.2919	0.2947	−0.0028	−0.95
291.39	0.2856	0.2864	0.2893	−0.0029	−1.00
293.14	0.3020	0.2817	0.285	−0.0033	−1.16
295.27	0.3223	0.277	0.2798	−0.0028	−1.00
297.55	0.3450	0.2771	0.2745	0.0026	0.95
299.65	0.3665	0.2674	0.2697	−0.0023	−0.85

Absolute average of relative deviations: 1.02%
[a] ν is the experimental result
[b] ν' is the literature value [27]

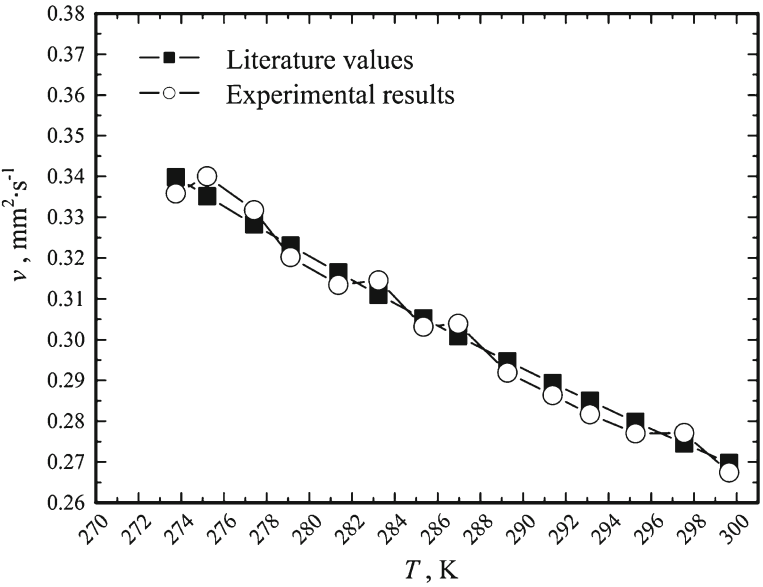

Fig. 15 Comparisons of the experimental results and literature values for R600a

viscosity substances such as water and refrigerants, the inner diameter of the glass tube is usually determined to be 2–3 mm. As for high viscosity substances such as oil, the inner radius is usually determined to be 3–5 mm but does not exceed 5 mm. The governing factor affecting the velocity of the liquid column is the component force of the liquid column's gravity along the axial direction of the pipe which is caused by the inclination angle of the glass tube. In the experimental process, whether there is

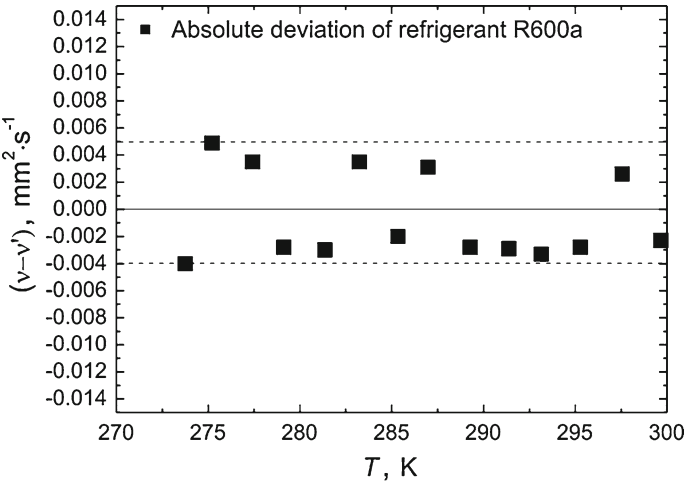

Fig. 16 Distribution of the absolute deviations for measurements of R600a

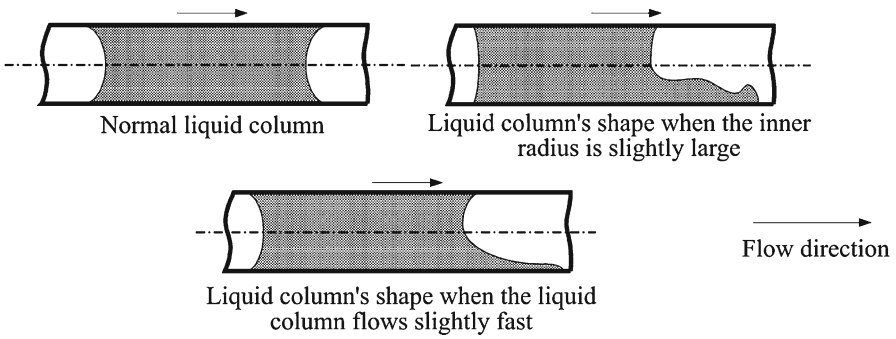

Fig. 17 Impact of glass tube's inner radius and liquid column's flow velocity on the shape of liquid column

an exceedingly large inclination angle can be judged according to the flow condition. If there is a large deviation among the three average velocities obtained by the four groups of optical fiber sensors, it can be concluded that there are large fluctuations in the flow of the liquid column and the inclination angle is too large. In the process of measurements, such as for ordinary substances, the initial inclination angle is usually determined to be 1–2° and the incremental step is 0.05°, but the inclination angle should not exceed 5°. It should be noted that in order to get good front and back meniscuses, as with a stable velocity of the liquid column, the inner wall of the glass tube should be fully wetted by the experimental liquid before measurement.

In comparison of the experimental results with standard values, it can be seen that there is an experimental measurement deviation which is about 1% using the inclined-tube viscometer.

1. There are many uncertainty factors impacting measurements of viscosity. From the literature, it can be seen that there are deviations between different data sources.

Data for the same substance, measured at the same conditions, are difficult to obtain from the literature; thus, the NIST database was chosen to be the reference for comparison.

2. As far as the measuring principle is concerned, it is necessary to perform some adjustments to improve the precision in future experimental studies.
3. As far as the experimental system is concerned, the uncertainty is largely determined by two factors. One factor is that in the measurements, the practical flow of the liquid column is not completely consistent with the ideal model described in Sect. 2. The other factor is that the measurement accuracy of the correlative parameter of the liquid column needs further improvement, especially the velocity measurement of the liquid column. As far as the constant temperature cabinet is concerned, due to the small specific heat capacity of the air, it is difficult to control the air in the chamber at the set temperature and to minimize the temperature fluctuations.

Compared with traditional capillary viscometry, the novel inclined-tube viscometry introduced in this article has the following advantages: 1. The measurement equation does not require a kinetic-energy correction and an end correction. 2. Measurement uncertainties caused by the surface tensions in the traditional capillary viscometer are minimized. 3. Due to the larger inner diameter of the experimental tube, which is more than 2 mm, the diameter effect in traditional capillary viscometry is considerably reduced. 4. It is straightforward and relatively simple to measure the viscosity at different temperatures and pressures.

7 Summary

In this article, a new method for liquid viscosity measurements is reported, the principle of the method is introduced, and the measurement equation is developed. Then a new experimental cell, which can be used to measure the viscosity of liquids at normal atmospheric pressure, that of liquids under high pressures, and for low-boiling substances in the liquid phase, and its corresponding data collection system and constant temperature cabinet were designed and developed. The reliability of the measuring principle and the uncertainty of the experimental system were tested and verified by measuring the kinematic viscosity of pure water at normal atmospheric pressure. The kinematic viscosities of refrigerants R134a and R600a in the saturated liquid phase were also measured. In this work, the experimental uncertainties in temperature, pressure, and viscosity measurements are estimated to be not greater than ± 0.24 K, ± 3.72 kPa, and $\pm 0.64\%$, respectively. In comparisons with literature data, the absolute average of relative deviations for pure water and refrigerants R134a and R600a are 0.84%, 1.04%, and 1.02%, respectively.

Acknowledgments This work was supported by the National Nature Science Fund Committee (NSFC No. 50376049), Program for New Century Excellent Talents in University (NCET-04–0925) and NSFC Fund for Creative Research Groups (No. 50521604). The authors gratefully acknowledge them for financial support of this work.

References

1. R.B. Bird, *Transport Phenomena* (John Wiley & Sons, New York, 2002), pp. 1–3
2. J. Kestin, W.A. Wakeham, *Transport Properties of Fluids*: *Thermal Conductivity*, *Viscosity*, *and Diffusion Coefficient* (Hemisphere, New York, 1988), pp. 75, 76
3. J.L. Cai, Sh.Q. Bo, R.Sh. Cheng, Colloid Polym. Sci. **282**, 182 (2003)
4. S. Shin, D.Y. Keum, J. Food Eng. **58**, 5 (2003)
5. A. Laesecke, T.O.D. Lüddecke, R.F. Hafer, D.J. Morris, Int. J. Thermophys. **20**, 401 (1999)
6. G. Mason, B.A. Buffham, M.J. Heslop, B. Zhangs, Chem. Eng. Sci. **63**, 2665 (1998)
7. H.Z. Chen, *Viscosity Measurement* (China Metrology Pub. House, Beijing, 2003), pp. 24–141 [In Chinese]
8. G.G. Liang, China Patent ZL 95115461.3, 30 Aug 2000 [In Chinese]
9. M. Madan, D. Mazumdar, Metall. Mater. Trans. B. **35**, 805 (2004)
10. S.K. Kawatra, A.K. Bakshi, Int. J. Miner. Process. **44–45**, 155 (1996)
11. R. Brust, V. Muth, J. Anal. Chem. **311**, 362 (1982)
12. M.R. Mustafaev, High Temp. **44**, 633 (2006)
13. M.R. Mustafaev, High Temp. **38**, 118 (2000)
14. N.A. Park, T.F. Irvine, Int. Comm. Heat Mass. **24**, 303 (1997)
15. N.A. Park, T.F. Irvine, Heat Mass Transfer **18**, 201 (1984)
16. F.J.P. Caetano, J.L. Correia da Mata, J.M.N.A. Fareleira, C.M.B.P. Oliveira, W.A. Wakeham, Int. J. Thermophys. **25**, 1 (2004)
17. F. Audonnet, A.A.H. Pádua, Fluid Phase Equilib. **181**, 147 (2001)
18. P.S. van der Gulik, Int. J. Thermophys. **16**, 867 (1995)
19. J. Wilhelm, E. Vogel, J.K. Lehmann, W.A. Wakeham, Int. J. Thermophys. **19**, 391 (1998)
20. S. Kim, Y. Cho, B. Hogenauer, K.R. Kensey, J. Non-Newtonian Fluid Mech. **103**, 205 (2002)
21. S. Kim, Y. Cho, A.H. Jeon, J. Non-Newtonian Fluid Mech. **94**, 47 (2000)
22. S. Shin, D.Y. Keum, Biosens. Bioelectron. **17**, 383 (2002)
23. H.Y. Yang, H. Li, P.P. Zhu, Polym. Test. **23**, 897 (2004)
24. R.Sh. Cheng, Y. Yang, Polymer **40**, 3773 (1999)
25. L.J. Yang, T.J. Yao, Y.Ch. Tai, J. Micromech. Microeng. **14**, 220 (2004)
26. A.P. Ershov, Z.M. Zorin, V.M. Starov, J. Colloid Interf. Sci. **216**, 1 (1999)
27. E.W. Lemmon, M.O. McLinden, M.L. Huber, NIST Reference Fluid Thermodynamic and Transport Properties—REFPROP Version 7.0 Users' Guide (National Inst. of Stands. Technol., Boulder, Colorado, 2002), pp. 61–78